International Journal of Engineering, Science and Mathematics
Vol. 6 Issue 8 Decembert2017, ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

ON GRAPH COMPLETION OF A GRAPH

DrAjitha V

Associate Professor

P.G Department of Mathematics

Mahatma Gandhi College, Iritty-670703

Kerala, India

Bibin K Jose

P.G & Research Department of Mathematics

S.D. College, Alappuzha

Kerala, India

Manju Raju

U.C. College Aluva

Kerala, India

Abstract

In this paper we establish the completion of graphs using degree sequences and establish some results. Also we introduce two types of indices related to this graph completion and shall attempt to compute these indices for some standard graphs.

Keywords

Completion of graphs, completion index type I, completion index type II.

1. Introduction

By a $(p,q)-$graph $G=\left(V(G),E(G)\ \right)$ we mean a finite undirected simple graph. The degree sequence of G is the list of vertex degree of G usually written in non increasing order $d_1 \geq d_2 \geq ... \geq d_n$. A graphic sequence is a list of non negative numbers that is the degree sequence of some simple graph. For various graph theoretic notations and terminology we follow F. Harrary [1] and D B West [2]. For basic number theoretic results we refer [3]. In this paper we consider simple graphs only.

In this article, we construct a new graph Comp(*G*) from a graph *G* as follows. The vertex set of Comp(*G*) is same as *G*and two vertices of Comp(*G*) are adjacent if and only if their degrees are equal in the graph *G*. Then the following theorem is immediate.

Theorem 1.1: Two vertices of Comp(*G*) are adjacent if and only if their degrees are equal in the graph *G*. This is an equivalence relation.

Proof: Since we consider simple graphs, we are not considering the loops, the reflexive property is taken as trivial. If *u* and *v* are adjacent in Comp(*G*) then *u* and *v* have same degree in *G*. Thus*v* and *u* are adjacent in Comp(*G*). Thus symmetry holds. For the transitivity, let *u* is adjacent to *v* and *v* is adjacent to *w* in Comp(*G*). Then *u* and *v* have same degree in *G* and also *v* and *w* have same degree in *G*. Thus*u* and *w* have the same degree in *G*. Hence *u* is adjacent to *w* in Comp(*G*). Hence it is an equivalence relation.

Thus a formal definition of the newly defined graph is as follows.

Definition 1.2: Givena $(p,q)-$graph G, one can define the completion of G as follows: corresponding to each vertex of G there is a vertex in the graph completion and two vertices of graph completion are joined by an edge if and only if their degrees are equal in the graph G. The graph completion is denoted by Comp(G).

Remark 1.3: Two vertices of Comp(*G*) are adjacent if and only if their degrees are equal in the graph *G*. This is an equivalence relation. Thus the vertices of Comp(*G*) are partitioned into different classes. Each vertex in a particular class is adjacent to all other vertices of the same class, since the degrees of the vertices are same in the class. Hence each class is isomorphic to a complete graph. Thus the most important fact is that Comp(*G*) is a disjoint union of complete graphs.

The following are some simple observations which follow immediately from the definition of graph completion.

Observation 1.4: Every graph *G* has a completion .

Observation 1.5: For a *k*-regular graph *G*, the completion is connected.

Observation 1.6: In general Comp(G) is a disjoined union of complete graphs.

Observation 1.7: If $G_1 \cong G_2$ then $\mathrm{Comp}(G_1) \cong \mathrm{Comp}(G_2)$

Remark 1.8: The converse of the above Observation need not be true. For example, the non-isomorphic graphs cycles, C_n and complete graphs, K_n give the same graph completion K_n.

Observation 1.9: Let *G* be a graph with degree sequence $d_1 \geq d_2 \geq ... \geq d_n$. Let *D* = {$d_1$, d_2, ...d_r} be the set formed by taking numbers of this degree sequence (Note that *D* does not contains repeated elements). Let the number d_i repeated n_i times in the degree sequences. Then the edges of Comp(G) is given by $\sum n_i C_2$ ($n_i \neq 0$ & 1).

Observation 1.10: From the degree sequence of G, we can find the clique number of Comp(G) as the largest repeating number in the degree sequence of the graph.

Observation 1.11: If Comp(G) contains K_n then G has *n* vertices with equal degrees.

Theorem 1.12: *G* and its complement graphhave same graph completion.

Proof: If the degree of the vertex *v*, deg(*v*) = *r* in G the deg(*v*) = *n*-*r*-1 in the complement of *G*. Thus, the degrees of vertices in the same equivalence classes have same degree in the complement also. Hence *G* and its complement graphhave same graph completion.

2. Completion of Certain Graphs

3. For star graphs $K_{1n}, \mathrm{Comp}(K_{1n}) = K_1 \cup K_n$

For Paths P_n, $\mathrm{Comp}(P_n) = K_2 \cup K_{n-2}$, for $n \geq 3$

For path P_2, Comp(P_2) = K_2

For *k*-regular graph *G* with *p* vertices Comp(G) = K_n

For Cycles, Comp(C_n) = K_n

For complete graphs Comp(K_n) = K_n

For Complete Bipartite graphs K_{mn}, Comp(K_{mn}) = $K_n \cup K_m$

For Wheels W_n, Comp(W_n) = $K_1 \cup K_{n-1}$

Theorem 2.1: The graph *G* is regular if and only if the graph completion, Comp(*G*) is isomorphic to the complete graph K_n..

Proof: Suppose that *G* is regular. Then all the vertices of Comp(*G*) are mutually adjacent. Hence, Comp(*G*) is isomorphic to $K_{n.}$. Conversely suppose that the graph completion, Comp(*G*) is isomorphic to the complete graph $K_{n.}$. Then, all the vertices of *G*has same degree and hence *G* is regular.

Theorem 2.2: The graph completion, Comp(*G*) is connected if and only if *G* is regular.

Proof: First suppose that Comp(*G*) is connected. If *G* is not regular, there exists vertices *u* and *v* such that *u* and *v*has distinct degrees. Thus*u* and *v* are not adjacent in Comp(*G*). Hence *u* and *v* lie in different components of Comp(*G*). Thus Comp(*G*) is disconnected , a contradiction.

Conversely, let *G* is regular. Then all the vertices of Comp(*G*) are mutually adjacent. Hence, Comp(*G*) is connected.

Theorem 2.3: There exists no graphs G such that the graph completion, Comp(*G*) is isomorphic to the totally disconnected graph.

Proof: If there exists a graph *G* with Comp(*G*) is totally disconnected. Then, all the elements of the degree sequence must be distinct. Thus, all the *n* vertices must have distinct degrees and hence the possible degrees are *n*-1, *n*-2, …, 2,1,0. This is not possible since, if a vertex has degree *n*-1, no other vertex in *G* has degree 0. Thus the above degree sequence does not exists in a graph *G*. Hence at least two nodes of the graph *u* and *v* have the same degree. Consequently these nodes *u* and *v* are connected in Comp(*G*). Hence Comp(*G*) is not totally disconnected.

Corollary 2.4: If Comp(*G*) = $K_1 \cup K_1 \cup \ldots \cup K_1$ then such a *G* does not exists.

4. Graphs Related to Disjoint Union of Complete Graphs

1. If Comp(G) = K_1 then $G = K_1$
2. If Comp(G) = $K_1 \cup K_1$ then G does not exist.
3. If Comp(G) = $K_1 \cup K_2$ then $G = P_3$ or $K_1 \cup K_2$
4. If Comp(G) = $K_1 \cup K_2 \cup K_2$ then $G = K_1 \cup P_4$

International Journal of Engineering, Science and Mathematics
Vol. 6 Issue 4, August 2017, ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Thus we shall find certain graphs whose completion graph is given by disjoint union of complete graphs. An exclusive study of this special class of graphs would provide scope for an independent direction of research, which we leave open at this stage. Thus for a given Comp(G), finding the graph G is not an easy task. For a given Comp(G), there may be many non-isomorphic graphs G exists or in some cases such G does not exists. So for a give set of numbers $\{p_1, p_2, \ldots, p_r\}$, if Comp($G$) = $K\,p_1 \cup K\,p_2 \cup \ldots \cup K\,p_r$, the problem of finding the graphs G is very interesting. The above examples shows that "not all such set of numbers" give Comp(G) for some graphs G. So finding such set of numbers is very interesting.

Problem 3.1: Find all Comp(G) with a unique graph G.

Problem 3.2: Does there exists a Comp(G) other than the totally disconnected graph such that the corresponding graph G does not exists?

5. Completion Indices Type I and Type I

From the above examples we conclude that sometimes completion graphs contains isolated vertices. We denote the number of isolated vertices of the graph completion by $\partial[\mathrm{Comp}(G)]$ and it is called the Index of Type I and the number of components of G is denoted by $\omega[\mathrm{Comp}(G)]$ and is called the Index of Type II. These two indices are mainly depends on the partition of $2q$ to get a graphic sequence.

Theorem 4.1: LetG be a graph with n vertices. Then $\partial[\mathrm{Comp}(G)] < \omega[\mathrm{Comp}(G)]$

Proof: By Theorem 2.3, Comp(G) is not a totally disconnected graph. Hence $\partial[\mathrm{Comp}(G)]$ $\neq n$. Hence $\partial[\mathrm{Comp}(G)] < \omega[\mathrm{Comp}(G)]$

Corollary 4.2: Let$G$ be a graph with n vertices. Then $\partial[\mathrm{Comp}(G)] <$ n.

Theorem 4.3: $\omega[\mathrm{Comp}(G)]$ =1 if and only if G is regular.

Proof: Clear from Theorem 2.1.

Theorem 4.4: Let Let$G$ be a graph and let $D = \{d_1, d_2, \ldots d_r\}$ be the set formed by taking numbers of this degree sequence. Then $\partial[\mathrm{Comp}(G)] = |D|$.

Problem 4.5: Find $\partial[\mathrm{Comp}(G)]$ and $\omega[\mathrm{Comp}(G)]$ for different classes of graphs

International Journal of Engineering, Science and Mathematics
Vol. 6 Issue 4, August 2017, ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Problem 4.6: Characterize (*p*,*q*)- graphs with $\omega\left[\text{Comp}(G)\right]$ and $\partial\left[\text{Comp}(G)\right]$ is *p*-1.

REFERENCES

[1] F. Harary Graph Theory, Addition-Wesley, Reading, Mass, 1972

[2] D B West, Introduction to Graph Theory, Prentice Hall, 2001.

[3] David M. Burton, Elementary Number Theory, Second Edition, Wm. C. Brown Company Publishers, 1980.

[4] J.A. Gallian, A Dynamic Survey of Graph Labeling, Electronic Journal of Combinatorics, 17 (2010), DS6.